Unifying Execution of Imperative and Declarative Code




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33rd International Conference on Software Engineering
May 27, 2011

Solving Sudoku

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	1					4	8	9
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Sudoku puzzle: fill in the empty cells s.t.:

1. all rows contain all values from 1 to 9
2. all columns contain all values from 1 to 9
3. all sub-grids contain all values from 1 to 9

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Approaches:

- write a custom (heuristic-based) algorithm [imperative]
- write a set of constraints and use a constraint solver [declarative]

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Sudoku with **Squander**

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public class Sudoku {
  private int[][] grid = new int[9][9];



  public void solve() { ??? }

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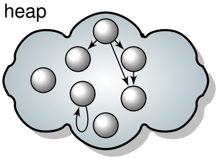


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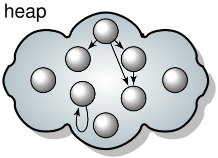


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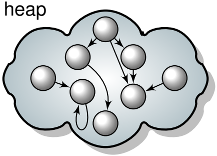
heap



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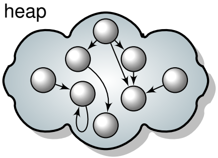

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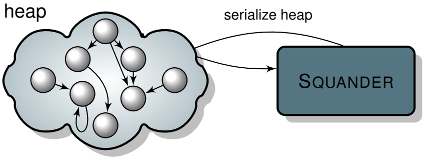


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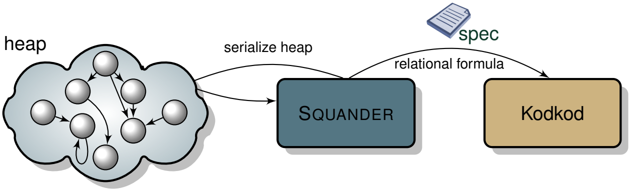


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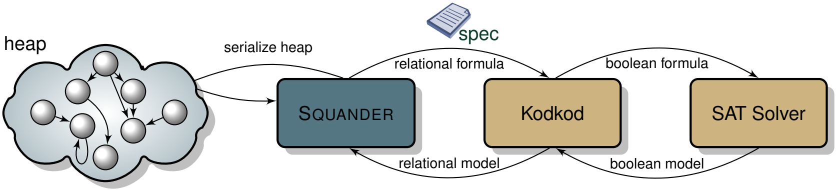


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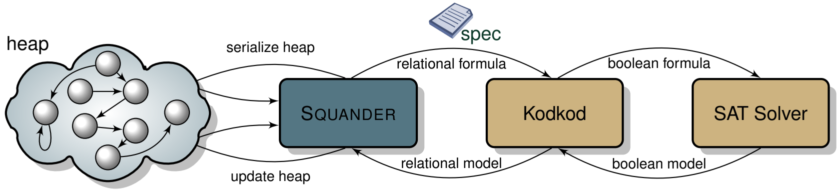


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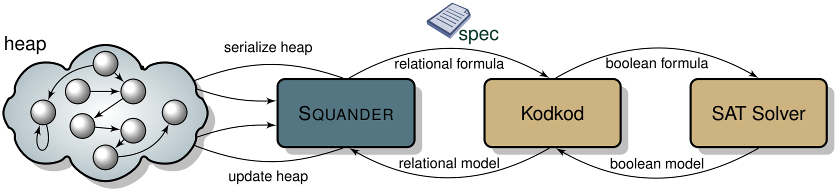


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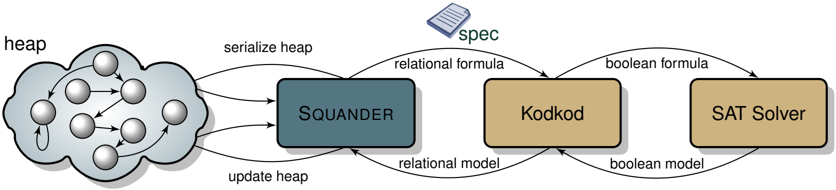


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executable first-order relational specifications for Java

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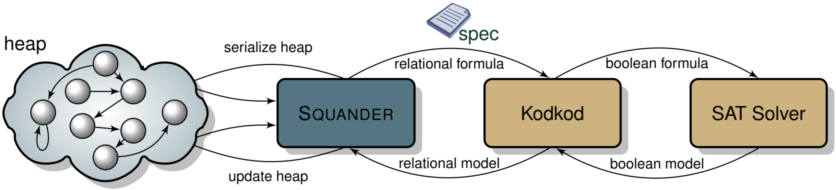


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specify and solve constraint problems in place

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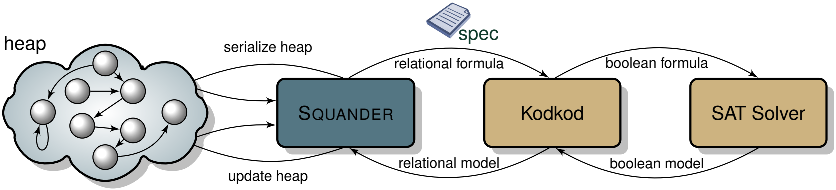


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specify and solve constraint problems in place

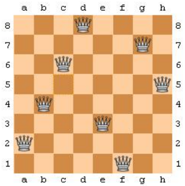
executable first-order relational specifications for Java

no manual translation to/from an external solver

SQUANDER vs Manual Search

N-Queens

- *place N queens on an N×N chess board such that no two queens attack each other*



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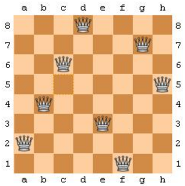

A backtracking with pruning solution

```
static boolean solveNQueens(int n, int col, int[] queenCols,
                            boolean[] bRow, boolean[] bD45, boolean[] bD135) {
  if (col >= n)
    return true;

  for (int row = 0; row < n; row++) {
    if (bRow[row] || bD45[row + col] || bD135[col - row + n - 1])
      continue;
    queenCols[col] = row;
    bRow[row]              = true;
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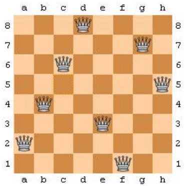

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doesn't look terribly bad, but fairly complicated

SQUANDER vs Manual Search

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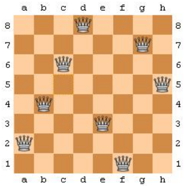

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how do you argue that it is correct?

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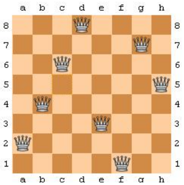

A solution with SQUANDER

```
@Ensures({
   "all disj q, r: result.elts |  " +   // for every two different queens q and r ensure that they are
   "   q.i          != r.i        && " +   //    not in the same row
   "   q.j          != r.j        && " +   //    not in the same column
   "   q.i − q.j != r.i − r.j  && " +   //    not in the same ↖↘ diagonal
   "   q.i + q.j != r.i + r.j" })          //    not in the same ↙↗ diagonal
@Modifies({
   "result.elts.i from {0 ... n−1}",      // modify fields i and j of all elements of
   "result.elts.j from {0 ... n−1}" })   // the result set, but only assign values from {0, ..., n−1}
static void solveNQueens(int n, Set<Queen> result) {
    Squander.exe(null, n, result);
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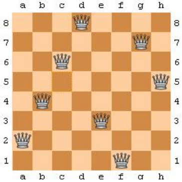

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  "   q.i − q.j  != r.i − r.j  && " +   //   not in the same ↘ diagonal
  "   q.i + q.j  != r.i + r.j" })       //   not in the same ↗ diagonal
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says what, not how

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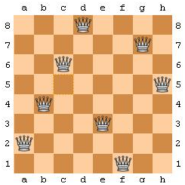

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  "all disj q, r: result.elts |  " +   // for every two different queens q and r ensure that they are
  "   q.i        != r.i        && " +   //    not in the same row
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@Modifies({
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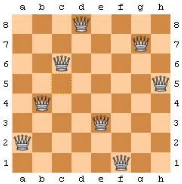
static void solveNQueens(int n, Set<Queen> result) {
    Squander.exe(null, n, result);
}
```

says what, not how

(almost) correct by construction!

SQUANDER vs Manual Search

N-Queens



- *place N queens on an N×N chess board such that no two queens attack each other*

A solution with SQUANDER

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What about performance?

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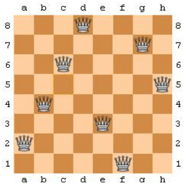

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says what, not how

(almost) correct by construction!

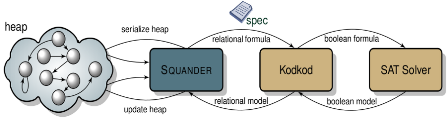
What about performance?

- It even outperforms the backtracking algorithm in this case!

Outline

Framework Overview

- specification language
- SQUANDER architecture



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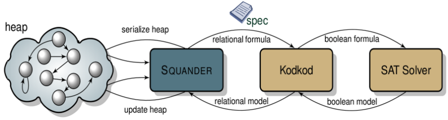
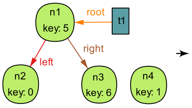


Translation

- from Java heap + specs to Kodkod
- minimizing the universe size



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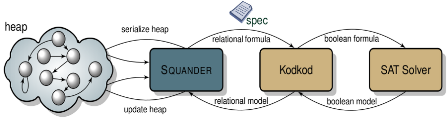

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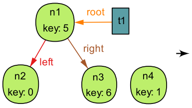

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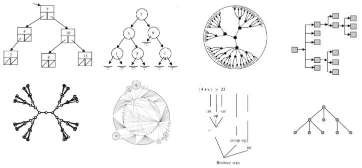
Treatment of Data Abstractions

- support for third party library classes (e.g. Java collections)



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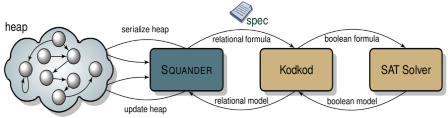

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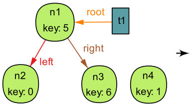

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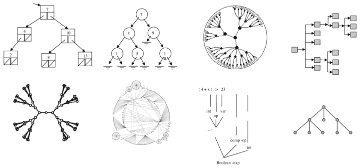
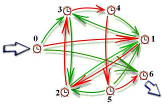
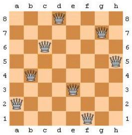


Evaluation/Case Study

- performance advantages for some puzzles and graph algorithms
- case study: *MIT course scheduler*





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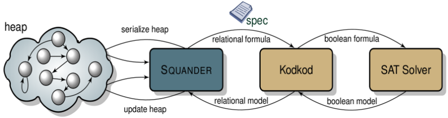

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Specification Language

Example - Binary Search Tree

```java
public class Tree {
  private Node root;
}
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```java
public class Node {
  private Node left, right;
  private int key;
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class specification field **@SpecField** ("<fld_decl> | <abs_func>")

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@Invariant({
  /* left sorted  */ "all x: this.left.*(left+right) − null | x.key < this.key",
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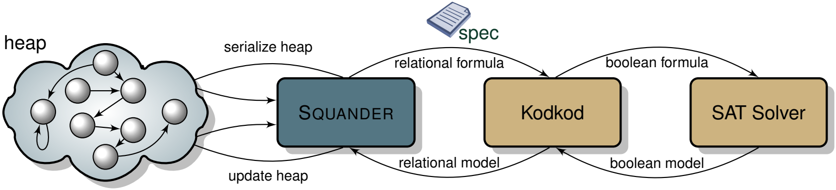

Execution steps

- traverse the heap and assemble the relevant constraints
- translate to Kodkod
  - translate the heap to relations and bounds
  - collect all the specs and assemble a single relational formula
- if a solution is found, update the heap to reflect the solution

Translation

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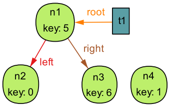

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From Objects to Relations

The back-end solver — Kodkod

- constraint solver for first-order logic with relations
- SAT-based finite relational model finder
  - finite bounds must be provided for all relations
- designed to be efficient for partial models
  - partial instances are encoded using bounds

From Objects to Relations

Translation of the `BST.insert` method

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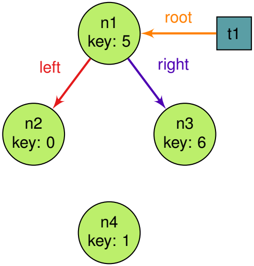

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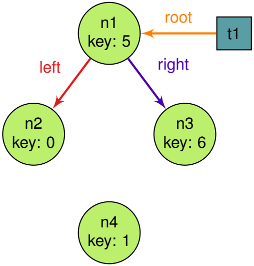

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reachable objects

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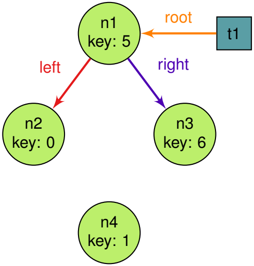

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pre-state

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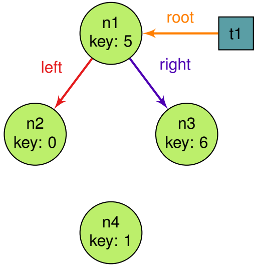

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pre-state

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post-state

lower bound   upper bound

- **lower bound**: tuples that must be included
- **upper bound**: tuples that may be included
- shrinking the bounds (instead of adding more constraints) leads to more efficient solving

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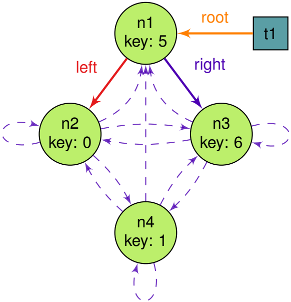

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pre-state

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root_pre:	$\{(t_1 \to n_1)\}$
left_pre:	$\{(n_1 \to n_2), (n_2 \to null), (n_3 \to null), (n_4 \to null)\}$
right_pre:	$\{(n_1 \to n_3), (n_2 \to null), (n_3 \to null), (n_4 \to null)\}$

post-state

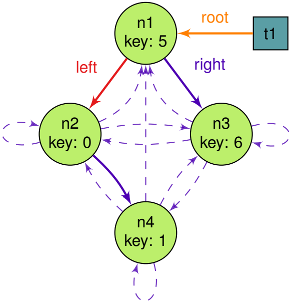
	lower bound	upper bound
root:	$\{\}$,	$\{t_1\} \times \{n_1, n_2, n_3, n_4, null\}$
left:	$\{n_1 \to n_2\}$,	$\{n_2, n_3, n_4\} \times \{n_1, n_2, n_3, n_4, null\}$
right:	$\{n_1 \to n_3\}$,	$\{n_2, n_3, n_4\} \times \{n_1, n_2, n_3, n_4, null\}$

- **lower bound**: tuples that must be included
- **upper bound**: tuples that may be included
- shrinking the bounds (instead of adding more constraints) leads to more efficient solving

From Objects to Relations

Translation of the `BST.insert` method

```
@Requires("z.key !in this.nodes.key")
@Ensures ("this.nodes = @old(this.nodes) + z")
@Modifies("this.root, this.nodes.left | _<1> = null, this.nodes.right | _<1> = null")
public void insertNode(Node z) { Squander.exe(this, z); }
```



reachable objects

BST1:	$\{t_1\}$	**N3**:	$\{n_3\}$	**BST_this**:	$\{t_1\}$
N1:	$\{n_1\}$	**N4**:	$\{n_4\}$	**z**:	$\{n_4\}$
N2:	$\{n_2\}$	**null**:	$\{null\}$	**ints**:	$\{0, 1, 5, 6\}$

pre-state

key_pre:	$\{(n_1 \to 5), (n_2 \to 0), (n_3 \to 6), (n_4 \to 1)\}$
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post-state

	lower bound	upper bound
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Performance of `Tree.insertNode`

What about performance now?

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- reason: tree insertion is algorithmically simple

  → imperative algorithm scales better than NP-complete SAT solving

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"Squander": *wasting CPU cycles for programmer's cycles*

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"Squander": *wasting CPU cycles for programmer's cycles*

Saving programmer's cycles

- **fast prototyping**: get a correct working solution early on
- **differential testing**: compare the results of imperative and declarative implementations
- **test input generation**: use SQUANDER to generate some binary trees

Generating Binary Search Trees with SQUANDER

```
@Ensures("#this.nodes = size")
@Modifies("this.root, Node.left, Node.right, Node.key")
@FreshObjects(cls=Node.class, num = size),
@Options(solveAll = true)
public void gen(int size) { Squander.exe(this); }
```

- to generate many different trees
  - the caller can use the SQUANDER API to request a different solution for the same specification

Treatment of Data Abstractions

Framework Overview

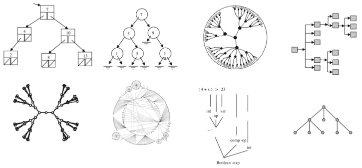
- specification language
- SQUANDER architecture

Translation

- from Java heap + specs to Kodkod
- minimizing the universe size

Treatment of Data Abstractions

- support for third party library classes (e.g. Java collections)



Evaluation/Case Study

- performance advantages for some puzzles and graph algorithms
- case study: *MIT course scheduler*

User-Defined Abstractions for Library Types

Why is it important to be able to specify library types?

- library classes are ubiquitous
- specs need to be able to talk about them

```
class Graph {
  class Node { public int key; }
  class Edge { public Node src, dest; }

  private Set<Node> nodes = new LinkedHashSet<Node>();
  private Set<Edge> edges = new LinkedHashSet<Edge>();

  // how to write a spec for the k-Coloring
  // problem for a graph like this?
  public Map<Node, Integer> color(int k) {
    return Squander.exe(this, k);
  }
}
```

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  }
}
```

- **solution**:
  - use `@SpecField` to specify abstract data types

User-Defined Abstractions for Library Types

How to support a third party class?

- write a spec file

```
interface Map<K,V> {
  @SpecField("elts: K -> V")

  @SpecField("size: one int | this.size = #this.elts")
  @SpecField("keys: set K | this.keys = this.elts.(V)")
  @SpecField("vals: set V | this.vals = this.elts[K]")

  @Invariant({"all k: K | k in this.elts.V => one this.elts[k]"})}
```

User-Defined Abstractions for Library Types

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```

- write an **abstraction** and a **concretization** function

```
public class MapSer implements IObjSer {
  public List<FieldValue> absFunc(JavaScene javaScene, Object obj) {
    // return values for the field "elts": Map -> K -> V
  }

  public Object concrFunc(Object obj, FieldValue fieldValue) {
    // update and return the given object "obj" from
    // the given values of the given abstract field
}}
```

Using Collections: Example

Now we can specify the k-Coloring problem

```
class Graph {
  class Node { public int key; }
  class Edge { public Node src, dest; }

  private Set<Node> nodes = new LinkedHashSet<Node>();
  private Set<Edge> edges = new LinkedHashSet<Edge>();

  @Ensures({
    "return.keys = this.nodes.elts",
    "return.vals in {1 ... k}",
    "all e: this.edges.elts | return.elts[e.src] != return.elts[e.dst]"})
  @Modifies("return.elts")
  @FreshObjects(cls = Map.class, num = 1)
  public Map<Node, Integer> color(int k) {return Squander.exe(this, k);}
}
```

```
interface Set<K> {
  @SpecField("elts: set K")

  @SpecField("size: one int |
    this.size=#this.elts")
}
```

```
interface Map<K,V> {
  @SpecField("elts: K -> V")

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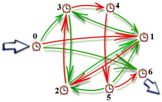
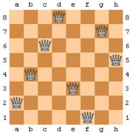
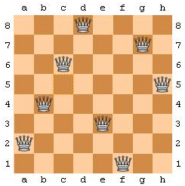


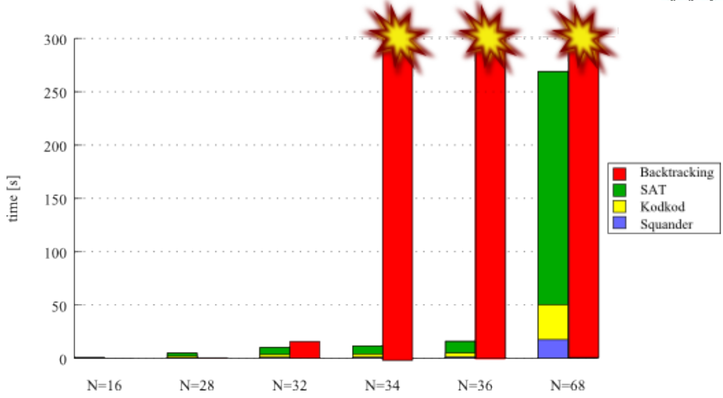


SQUANDER vs Manual Search

N-Queens

- place N queens on an N×N chess board such that no two queens attack each other





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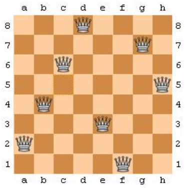
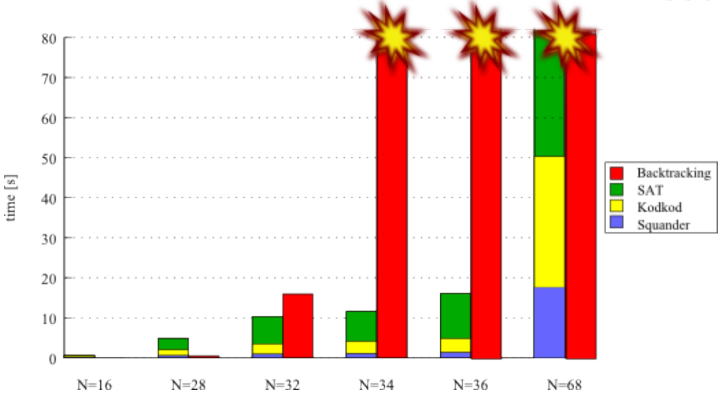
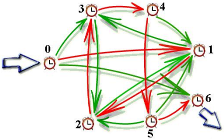




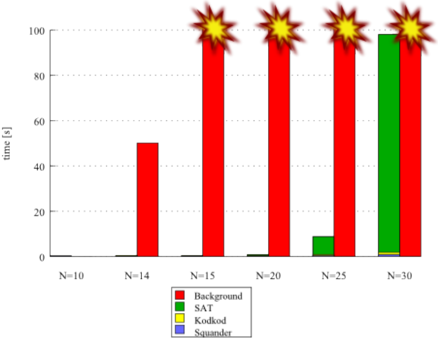
SQUANDER vs Manual Search

Hamiltonian Path

- find a path in a graph that visits all nodes exactly once



Graphs with Hamiltonian path



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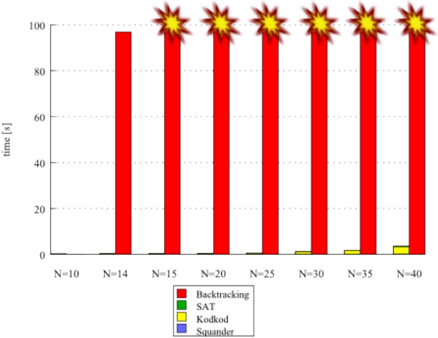

SQUANDER vs Manual Search

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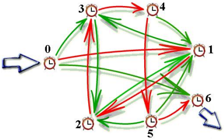

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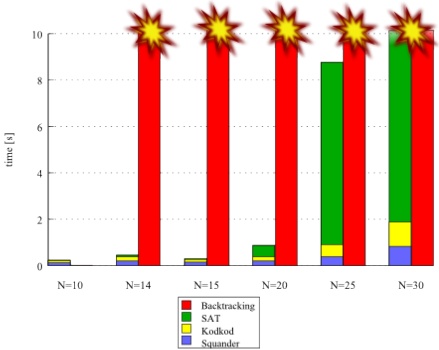

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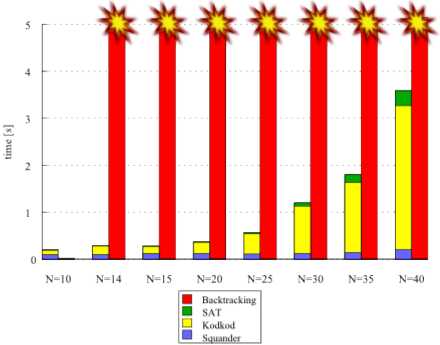


SQUANDER vs Manual Search

So, is SQUANDER always better than backtracking?

- **of course not!**

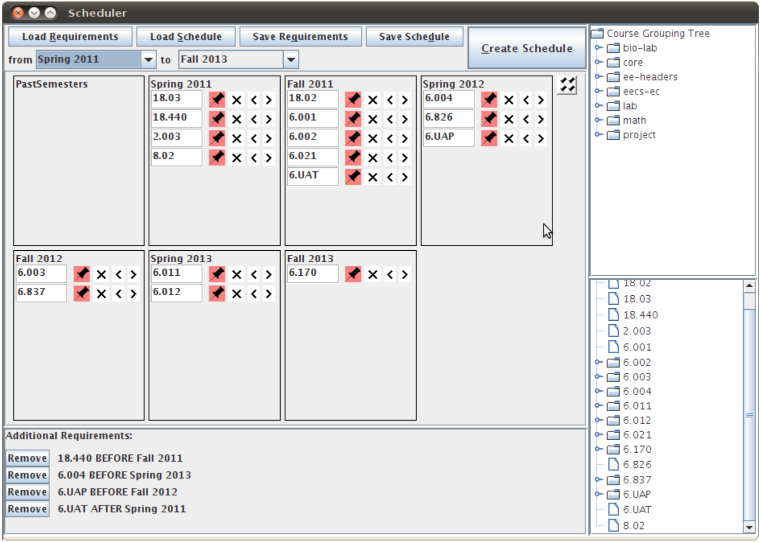
Rather, the takeaway point is

- if the problem is easy to specify, it makes sense to do that first
  1. you'll get a correct solution faster
  2. if the problem is algorithmically complex, the scalability might be satisfying as well

Other Evaluation Questions

- **usability on a real-world constraint problem**
- **annotation overhead**
- **ability to handle large program heaps**
- **efficiency**

Case Study – Course Scheduler



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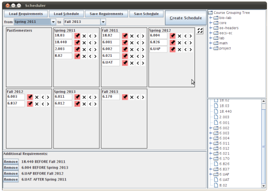

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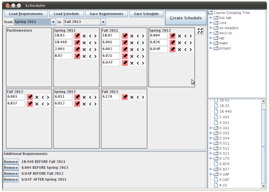

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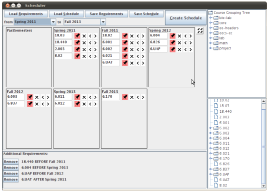

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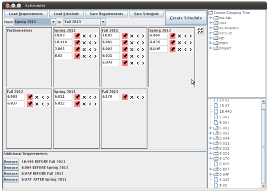

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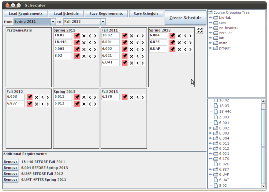

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- **ability to handle large program heaps**
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- **efficiency**
  - about 5s as opposed to 1s of the original implementation

Limitations

- **boundedness** – SQUANDER can't generate an arbitrary number of new objects; instead the maximum number of new objects must be explicitly specified by the user
- **integers** – integers must also be bounded to a small bitwidth
- **equality** – only referential equality can be used (except for strings)
- **no higher-order expressions** – e.g. can't specify *find the longest path in the graph*; instead must specify the minimum length k, i.e. *find a path in the graph of length at least k nodes*
- **debugging** – if a solution cannot be found, the user is not given any additional information as to why the specification wasn't satisfiable

Future Work

- **optimize translation** to Kodkod
  - use fewer relations to represent the heap
    (short-circuit some unmodifiable ones)
- **support debugging** better
  - when no solution can be found, explain why
    (with the help of unsat core)
- **synthesize** code from specifications
  - especially for methods that only traverse the heap
- **combine different solvers** in the back end
  - SMT solvers would be better at handling large integers

Summary

SQUANDER lets you

- execute first-order, relational specifications in Java

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Why would you want to do that?

- conveniently express and solve algorithmically complicated problems using declarative constraints
- gain performance in certain cases (e.g. for NP-hard problems)
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  - fast prototyping (get a correct working solution fast)
  - generate test inputs
  - runtime assertion checking

CSAIL

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Thank You!

http://people.csail.mit.edu/aleks/squander

CSAIL

Solving Sudoku with **Alloy Analyzer**

```
abstract sig Number {}
one sig N1,N2,N3,N4,N5,N6,N7,N8,N9 extends Number {}

one sig Global {
    data: Number -> Number -> one Number
}

pred complete [rows:set Number, cols:set Number]{
   Number = Global.data[rows][cols]
}

pred rules {
    all row: Number { complete[row,Number] }
    all col: Number { complete[Number,col] }
    let r1=N1+N2+N3, r2=N4+N5+N6, r3=N7+N8+N9 |
      complete[r1,r1] and complete[r1,r2] and complete[r1,r3] and
      complete[r2,r1] and complete[r2,r2] and complete[r2,r3] and
      complete[r3,r1] and complete[r3,r2] and complete[r3,r3]
}

pred puzzle {
   N1->N4->N1 + N1->N8->N9 +
   ...
   N9->N2->N2 + N9->N6->N1 in Global.data
}

run { rules and puzzle }
```

			1				9	
	6	7	9	2			4	5
				7	3	2		
	1					4	8	9
	7						5	
4	3	6					2	
		1	7	9				
7	4			3	2	9	1	
	9				1			

Solving Sudoku with **Kodkod**

```java
public class Sudoku {
  private Relation Number = Relation.unary("Number");
  private Relation data = Relation.ternary("data");
  private Relation[] regions = new Relation[] {
    Relation.unary("Region1"),
    Relation.unary("Region2"),
    Relation.unary("Region3") };

  public Formula complete(Expression rows, Expression cols) {
    // Number = data[rows][cols]
    return Number.eq(cols.join(rows.join(data))); }

  public Formula rules() {
    // all x,y: Number | lone data[x][y]
    Variable x = Variable.unary("x");
    Variable y = Variable.unary("y");
    Formula f1 = y.join(x.join(data)).lone().
      forAll(x.oneOf(Number).and(y.oneOf(Number)));
    // all row: Number | complete[row, Number]
    Variable row = Variable.unary("row");
    Formula f2 = complete(row, Number).
      forAll(row.oneOf(Number));
    // all col: Number | complete[Number, col]
    Variable col = Variable.unary("col");
    Formula f3 = complete(Number, col).
      forAll(col.oneOf(Number));
    // complete[r1,r1] and complete[r1,r2] and complete[r1,r3] and
    // complete[r2,r1] and complete[r2,r2] and complete[r2,r3] and
    // complete[r3,r1] and complete[r3,r2] and complete[r3,r3]
    Formula rules = f1.and(f2).and(f3);
    for(Relation rx: regions)
      for(Relation ry: regions)
        rules = rules.and(complete(rx,ry));
    return rules;
  }

  public Bounds puzzle() {
    Set<Integer> atoms = new LinkedHashSet<Integer>(9);
    for(int i = 1; i <= 9; i++) { atoms.add(i); }
    Universe u = new Universe(atoms);
    Bounds b = new Bounds(u);
```

			1				9	
	6	7	9	2			4	5
				7	3	2		
	1					4	8	9
	7						5	
4	3	6					2	
		1	7	9				
7	4			3	2	9	1	
	9				1			

```java
    TupleFactory f = u.factory();
    b.boundExactly(Number, f.allOf(1));
    b.boundExactly(regions[0], f.setOf(1, 2, 3));
    b.boundExactly(regions[1], f.setOf(4, 5, 6));
    b.boundExactly(regions[2], f.setOf(7, 8, 9));

    TupleSet givens = f.noneOf(3);
    givens.add(f.tuple(1, 4, 1));
    givens.add(f.tuple(1, 8, 9));
    ...
    givens.add(f.tuple(9, 6, 1));
    b.bound(data, givens, f.allOf(3));
    return b;
  }

  public static void main(String[] args) {
    Solver solver = new Solver();
    solver.options().setSolver(SATFactory.MiniSat);
    Sudoku sudoku = new Sudoku();
    Solution sol = solver.solve(sudoku.rules(), sudoku.puzzle());
    System.out.println(sol);
  }
}
```

Mixing Imperative and Declarative with SQUANDER

			1				9	
	6	7	9	2			4	5
				7	3	2		
	1					4	8	9
	7						5	
4	3	6					2	
		1	7	9				
7	4			3	2	9	1	
	9				1			

Mixing Imperative and Declarative with SQUANDER

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static class Cell { int num = 0; } // 0 means empty

@Invariant("all v: int - 0 | lone {c: this.cells.vals | c.num = v}")
static class CellGroup {
  Cell[] cells;
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			1				9	
	6	7	9	2			4	5
				7	3	2		
	1					4	8	9
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public class Sudoku {
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  public Sudoku(int n) {
      // (1) create CellGroup and Cell objects,
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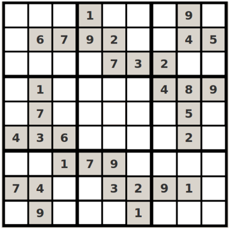

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    ...
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    s.solve();
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Write more imperative code to make constraints simpler

Everything is a relation

Everything is a relation

				relation name	relation type
classes	$\rightsquigarrow$ unary relations	**class** C {}	$\rightsquigarrow$	$\mathscr{R}_C$	: C
objects	$\rightsquigarrow$ unary relations	**new** C();	$\rightsquigarrow$	$\mathscr{R}_{C_1}$	: C
fields	$\rightsquigarrow$ binary relations	**class** C { A fld ; }	$\rightsquigarrow$	$\mathscr{R}_{fld}$	: $C \rightarrow A \cup \{$**null**$\}$
arrays	$\rightsquigarrow$ ternary relations	T[]	$\rightsquigarrow$	$\mathscr{R}_{T[]\_elems}$	: $T[] \rightarrow$ **int** $\rightarrow T \cup \{$**null**$\}$

Minimizing the Universe Size

Relations in Kodkod

$$r_k \xrightarrow{\text{in Kodkod}} M_{|univ| \times |univ| \times \cdots \times |univ|}$$

a relation of arity k

a matrix of dim $|univ|^k$

Minimizing the Universe Size

Relations in Kodkod

$$\underset{\text{a relation of arity } k}{r_k} \xrightarrow{\text{in Kodkod}} \underset{\text{a matrix of dim } |univ|^k}{M_{|univ| \times |univ| \times \cdots \times |univ|}}$$

so

if $|univ| > 1291 \land (\exists_{r_k} \mid k \geq 3)$

Minimizing the Universe Size

Relations in Kodkod

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if $|univ| > 1291 \wedge (\exists_{r_k} \mid k \geq 3)$

$\implies dim(M) > 1291^3 = 2151685171 > \text{Integer.MAX\_VALUE}$

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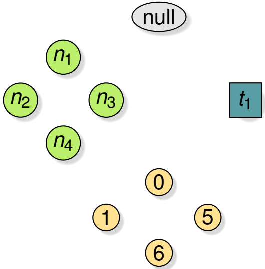
$\implies$ can't be represented in Kodkod

- ternary relations are not uncommon in SQUANDER (e.g. arrays)
- *MIT course scheduler* case study: almost 2000 objects
- **solution**:
  - partitioning algorithm that allows atoms to be shared

Minimizing the Universe

goal: use fewer Kodkod atoms than heap objects



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$\rightarrow$ multiple objects must map to same atoms
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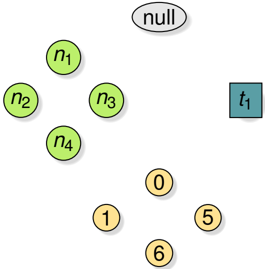

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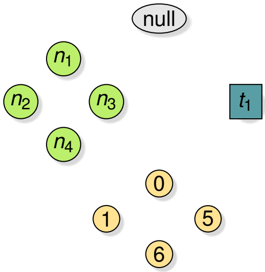

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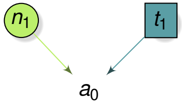

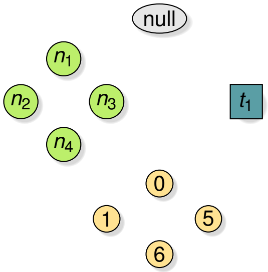

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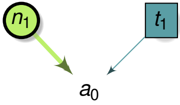

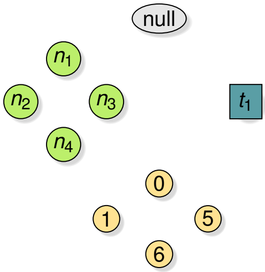

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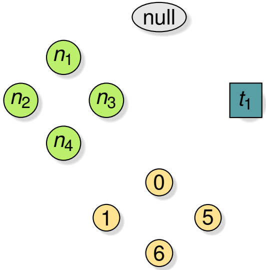

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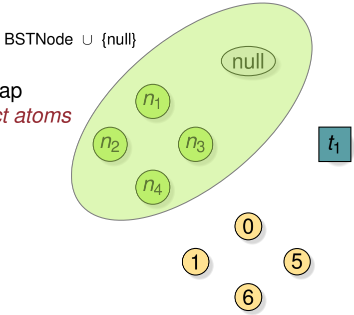

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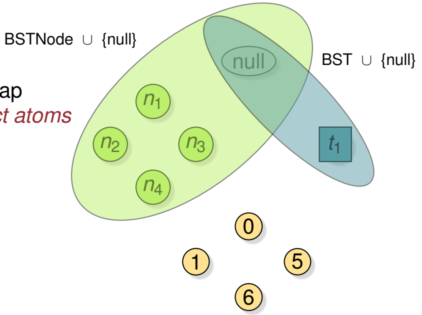

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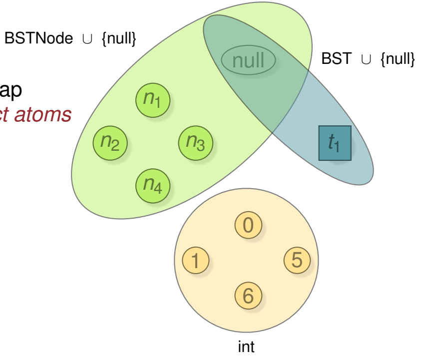

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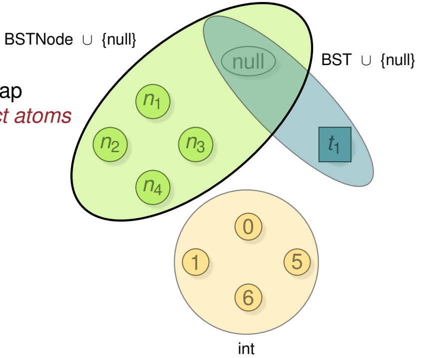

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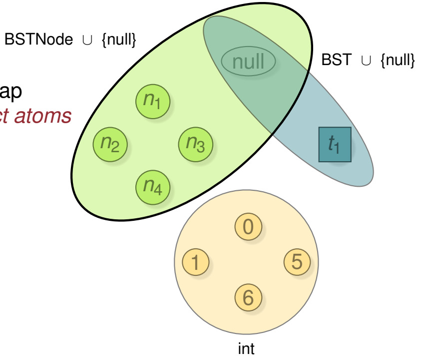

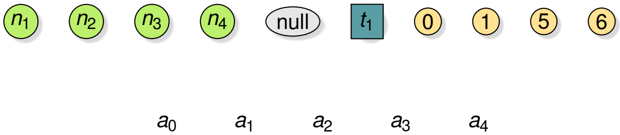

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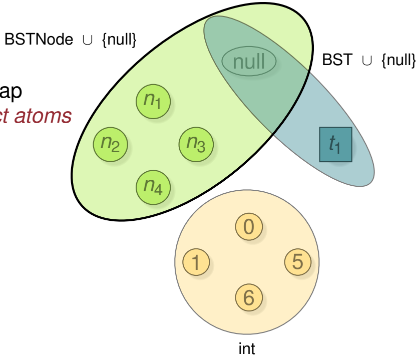

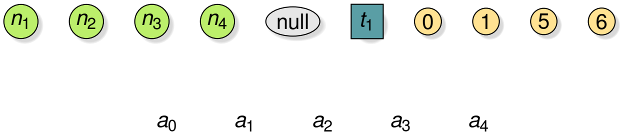

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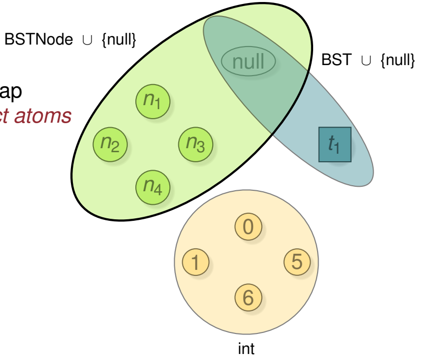

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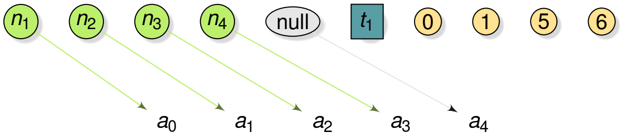

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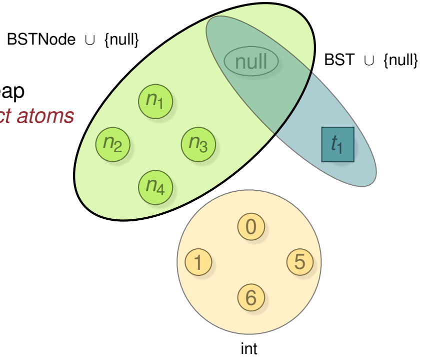

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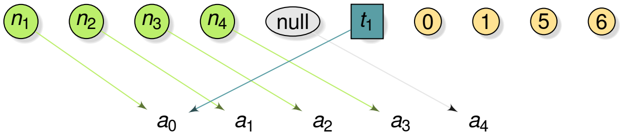

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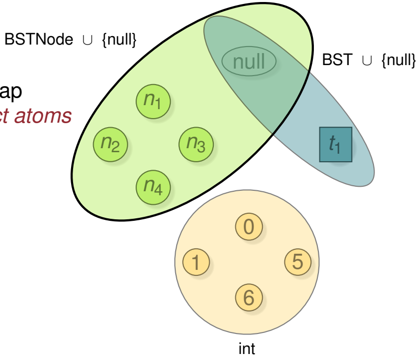

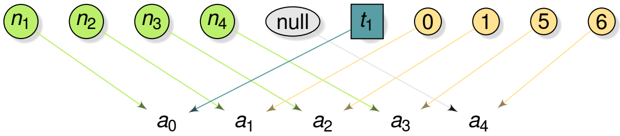

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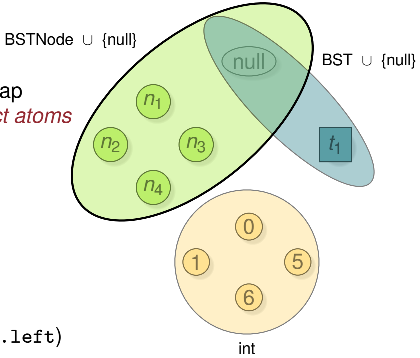

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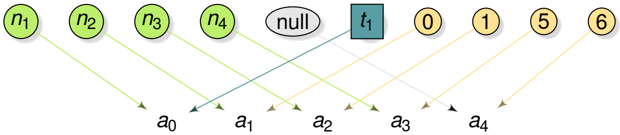

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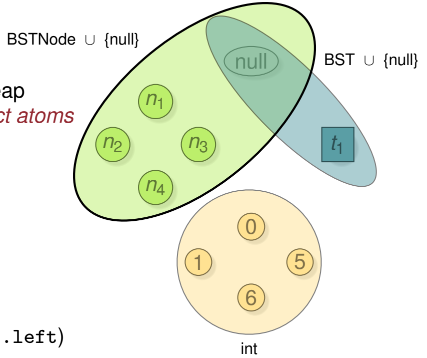

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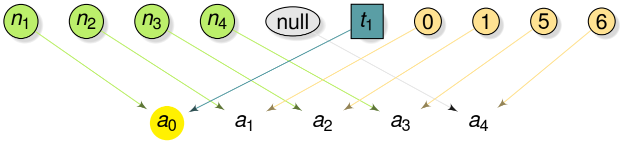

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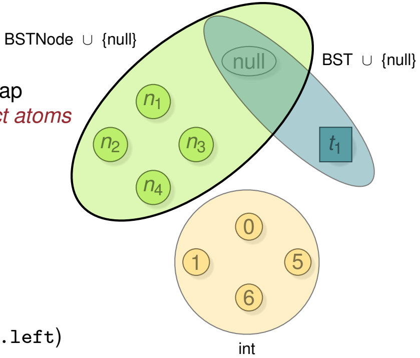

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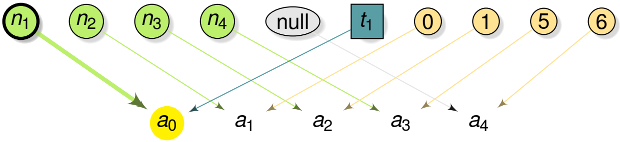
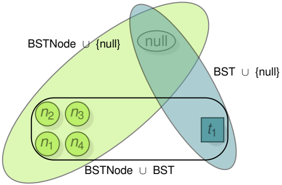


Partitioning Algorithm – Discussion

Why is this algorithm sufficient?

- what if we had partitions like this:



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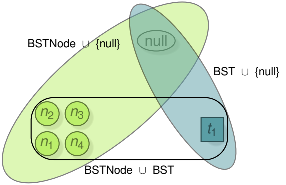

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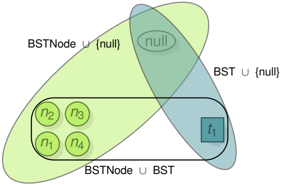

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Limitations

- no performance gain

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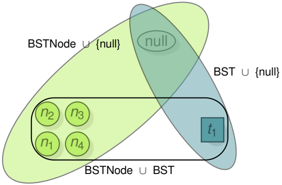

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- **but**, SQUANDER type checker disallows types like BSTNode ∪ BST

- or a spec like:

"no BSTNode & int"

- if nodes and ints shared atoms, then the intersection would not be empty!
- **again**, in Java, such expressions don't make much sense, so SQUANDER disallows them.

Limitations

- no performance gain
- if a field of type Object is used, this algorithm has no effect
  - everything is a subtype of Object so everything has to go to the same partition

Related Work

Executable Specifications:

- **An Overview of Some Formal Methods for Program Design**, C.A.R. Hoare (*IEEE Computer 1987*)
- **Specifications are not (necessarily) executable**, I. Hayes et al. (*SEJ 1989*)
- **Specifications are (preferably) executable**, N.E. Fuchs (*SEJ 1992*)
- **Programming from Specification**, C. Morgan, PrenticeHall, 1998
- **Agile Specifications**, D. Rayside et al. (*Onward! 2009*)
- **Falling Back on Executable Specifications**, H. Samimi et al. (*ECOOP 2010*)
- **Unified Execution of Imperative and Declarative Code**, A. Milicevic et al. (*ICSE 2011*)

Specification Languages

- **JFSL: JForge Specification Language**, K. Yessenov, MIT 2009
- **Software Abstractions: Logic, Language, and Analysis**, D. Jackson, MIT Press 2006

Programming Languages with Constraint Programming:

- **Jeeves: Programming with Delegation**, J. Yang, MIT, 2010
- **Programming with Quantifiers**, J.P. Near, MIT, 2010